**CSC148 – Quiz#3 – Tuesday Nov 5, 2019, Name** \_\_\_\_\_Answers\_\_\_\_\_ maxScore: 20\_\_\_\_\_

Closed notes; no mobile devices/calculators can be in-use during quiz

*An Appendix of terms and definitions at the end of this document can be used/referenced*

*Questions with phrase <show all work> require writing out and explaining steps (not just writing down a numeric answer); otherwise, credit loss will result*

**Q#1**. 4 pts In an M/M/1 Kendall system, with no specific t.u. (seconds/minutes/etc.) specified, let  = 1. Calculate the smallest service rate  so that the the ss avg(number of cj is S) is < 1. <*show all work and underline you answer*>.

L < 1 is given.  = , and since  is given as 1, .

In M/M/1, L =  and substituting for  gives () / ( 1 – ( ) < 1 and simplifying the inequality gives

2 \* ( < 1, so 2 < . Thus,  can be no smaller than 2

**Q#2.** Given an M/M/3/10/10 system. Each cj never leaves S. The calling population members cj continuously repeat the state sequence:

in S and doing various activities {cj is in state ‘activity’}

queued for service {cj is in state ‘queued’}

obtaining service {cj is in state ‘service’}

A ) 2 pts If all servers are busy, and 4 of the cj are in the calling population (in state ‘activity’), calculate the number of cj in state ‘queued’

S always has the same total number of cj, but the cj are divided into: getting service, queued for service, and active.

We have the equation 10 = 3 + numberQueued + 4, so numberQueued = 3

B ) 1 pt If  is the arrival rate for S, calculate an expression (in terms of  and perhaps constants) for e the effective arrival rate when S is in the state for part A ).

By definitione at any instant is the % of the given  that is currently in the calling population. When S is in state of part A ), there are 4 in the calling population, so e = 0.4\*

**Q#3** Given an M/G/∞ system S,

A ) 2 pts What is the ss value of ? Hint: interpret the formula for utilization in the limiting situation where

the degree is ∞ (that is, unlimited).

Theoretically, we can have infinitely many servers. But this is approximated by specifying a very large value for the Kendall parameter c.

When c is finite, no matter how large, we have proved that  =  / (c\*). When c -- > ∞ the denominator of the ratio - - > 0, so - - > 0

(Another way to view this is: r = (avg number of busy servers) /(total number of servers) - -> (avg number of busy servers) /∞ - -> 0)

B ) Using only the terms Pk and possibly some constants, write an expression for the probability that there are at most 4 cj in S.

Notes – you do not need to know (nor evaluate) the known formula for Pk.

Let x = the unknown probability. There are at most 4 cj in S when there are 0 or 1 or 2 or … or 4 cj in S. The Pk are independent probabilities (there cannot be two different populations in S at any instant), so x = (P0+P1+P2+P3+P4)

Q#4 4 pts In a distribution fitting project for a sample data set “d”, an event in d is not a vehicle arrival; instead, an event is a washing machine/dryer service call into a nation-wide service/repair center. Random variable “x” is the number of calls occurring in an arbitrary hour. For the limited time in this quiz, d is specified as the following small frequency table:

|  |  |
| --- | --- |
| Possible number of calls in a random hour | Call frequency |
| 0 | 1 |
| 1 | 3 |
| 2 | 4 |
| 3 | 1 |
|  |  |

A ) 2 pts What is x^ = the average/mean number of calls per hour?

(0\*1 + 1\*3 + 2\*4 + 3\*1)/(number of observations) = (0\*1 + 1\*3 + 2\*4 + 3\*1)/(1+3+4+1) = 14/9 = 1.555…

B ) 2 pts If a data analyst is fitting d to a Poisson distribution, calculate the theoretical probability that 2 calls occur in any random hour.

Express your final answer as a number in the form: someRealNumber \* esomePower  = (14/9)2 /2! \* e-(14/9) for X=xi=2

SOME Python practice

Number of Classes: 4

Mean of the class frequencies: 1.56

sum of the class frequencies: 9

Chi-square class values calculations tool

The pmf values for pi = prob(X=xi), xi=0,1,2, ..., assuming NON-combined classes

p0 is: 0.21013607120076472

p1 is: 0.327812271073193

p2 is: 0.25569357143709054

p3 is: 0.13296065714728708

Poisson pmf, evaluated at X=xi, coded in Python3:

(math.exp(-OiFrequencyMean)\*OiFrequencyMean\*\*xi )/math.factorial(xi)

**#5.** 5 pts

A ) A bottleneck is a server with the largest utilization in a system.

B ) Arrival rate inbound into S has rate 1, and the rate into sc1 is 1/3 so the rate into sc2 is 2/3. At all sck in this problem, the Jackson open network assumes that arrival rate into an sck equals the departure rate from sck. This means that sc3 has arrival rate 1/3 + 2/3 = 1. Applying the formula for  to all sck (and using the correct degree for each sck) gives:

sc1 and sc2 both have r = 1/9, but sc3 utilization is 1/12, so sc1 and sc2 are both bottlenecks in S (and sc3 is not a bottleneck).